SEMI-MARKOV ANALYSIS OF A COMMUNICATIONS SYSTEM

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THESIS

SEMI-MARKOV ANALYSIS

OF A COMMUNICATIONS SYSTEM

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of a Communications System

by

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ABSTRACT

A communications system composed of n transmitters and r receivers and which is subject to failure is modeled as a semi-Markov process having four states. Transient and steady state results are analyzed to determine the probability that the system is in any given state. Two examples are given to illustrate the results in the transient as well as in the steady state.

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I. INTRODUCTION

A communications system is analyzed which has n transmitters and r receivers ($r < n$). Each transmitter works in cycles of transmission and idle periods of time, which are random. Each of the receivers is only capable of handling one message at the time, and therefore, messages transmitted while all r receivers are busy are then lost.

Transmitters are subject to failure during transmission and idle periods, and when this occurs, it takes a random amount of time to be repaired and put back in service.

The purpose of this paper is to describe a mathematical model for the system and analyze the transient and steady state results in order to determine the probability of a given message being lost.

The paper is written in four parts. In Part II a model of a Semi-Markov process is formulated and its variables and properties defined. Part III contains the transient results, and Part IV the steady state results.

II. MODEL FORMULATION

Given that the n transmitters composing the system have the same characteristics and work under the same conditions, the probability that any of them is transmitting at some arbitrary point in time is essentially the same.

Suppose there are only r receivers, each of which can receive only one message at the time. Then if L(t) is the probability that a message transmitted at t is lost,

$$L(t) = P\left[(r+1) \text{ or more messages are being transmitted at } t\right]$$

$$= \sum_{k=r+1}^{n} \binom{n}{k} p(t)^{k} \left[1-p(t)\right]^{k} ,$$

where p(t) is the probability an individual message is being transmitted at t.

Our problem is to determine p(t). Consider a system where a transmitter transmits for some fixed time α , and is idle (not transmitting) for some fixed time β . Also assume the transmitter never fails and that we measure time zero from the start of a new transmission time. Then,

$$p(t) = 1 \quad n(\alpha+\beta) \leq t \leq n(\alpha+\beta) + \alpha, \; n = 0,1,2,\ldots$$

$$= 0 \quad \text{otherwise} .$$

This is illustrated in figure 1.

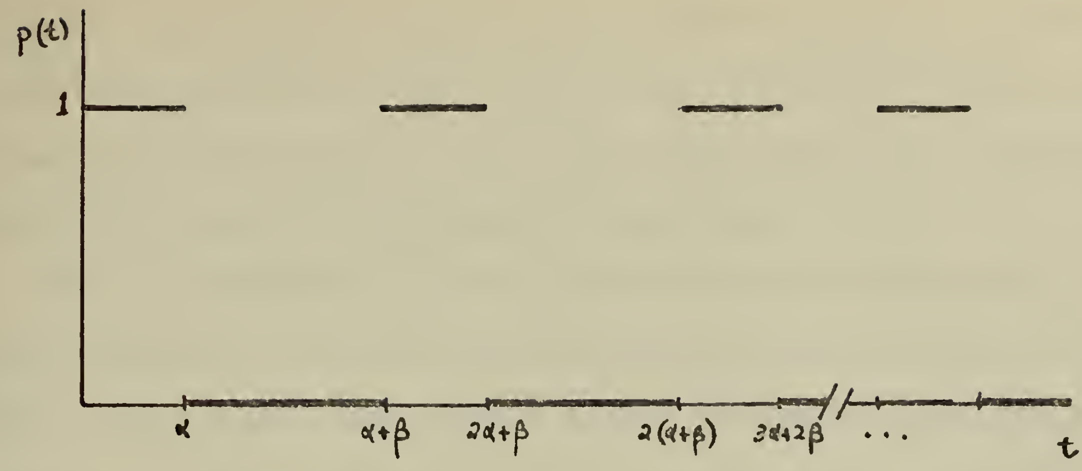

Figure 1. A simple example for p(t).

In our model we consider the transmission and idle times as random variables. Also we include the possibility of transmitter failure both while transmitting and while idling. We assume that if it fails, it takes a random repair time to fix, at which point it starts over a new transmit or idle time, depending on its state at time of failure. Thus the system can be in one of four states:

State	Description
0	Transmitting
1	Idle and up
2	On repair (failure occurred during tranmission)
3	On repair (failure occurred during idle period)

Let T_i be the length of the i^{th} transmission, $\{T_i\}$ i.i.d. random variables with distribution function (d.f.) T(t). Thus $T(t) = P(T_i \leq t)$, $i = 1,2...$. Also let I_i be the length of the i^{th} idle time, $\{I_i\}$ i.i.d. random variables with d.f. I(t). If the system is operating (either

transmitting or idle) we assume an exponential time to failure with rate λ (mean time to failure is $1/\lambda$). Once failed, the time to repair is also assumed to be exponential at rate μ (mean repair time $1/\mu$) independent of whether the transmitter was idle or transmitting.

Thus the transmitter follows a stochastic process which makes transitions according to a Markov Chain when viewed at points at which it changes state. It is crucial that times to failure be exponential for the Markov property to hold. For example, if the transmitter makes a transition from idle to transmit (state 1 to state 0), then if the distribution of time to the next transition is to be independent of all previous history of the process, the knowledge of when the system last failed and was repaired must be irrelevant. This can only be true with exponential times to failure. It is not important in the model that the repair times be exponential.

The amount of time spent in each state before a transition occurs is random. Defining $Z(t) = [\text{state of the process at time } t]$; then the stochastic process $\{Z(t),\ t \geq 0\}$ is a Semi-Markov Process, and $p(t) = P[Z(t) = 0]$.

We now define:

$$Q_{ij}(t) = P\left[\begin{array}{l}\text{process next makes a transition}\\ \text{into state } j \text{ and this occurs}\\ \text{in a time} \leq t\end{array} \middle| \begin{array}{l}\text{process has}\\ \text{just entered}\\ i\end{array}\right] \qquad (2)$$

$$P_{ij}(t) = P\left[\text{state at time } t \text{ is } j \middle| \begin{array}{l}\text{process started}\\ \text{in state } i \text{ at}\\ \text{time } 0\end{array}\right]$$

$$= P\left[Z(t) = j \mid Z(0) = i\right] \qquad (3)$$

$$F_{ij}(t) = P\left[\begin{array}{l}\text{a transition will occur}\\ \text{in an amount of time} \leq t\end{array} \middle| \begin{array}{l}\text{process has just}\\ \text{entered } i \text{ and}\\ \text{will next enter } j\end{array}\right] \qquad (4)$$

$$H_{ij}(t) = P\left[\begin{array}{l|l}\text{next transition occurs} & \text{process has} \\ \text{in an amount of time} \leq t & \text{just entered} \\ & i\end{array}\right] \tag{5}$$

$$P_{ij} = Q_{ij}(\infty) = P\left[\begin{array}{l|l}\text{next transition will} & \text{process just} \\ \text{be into state } j & \text{entered } i\end{array}\right]. \tag{6}$$

Since $Q_{ij}(t)$ is the joint probability that the next transition is from i to j and in time less than t, we have

$$F_{ij}(t) = \frac{Q_{ij}(t)}{P_{ij}} \qquad \text{when } P_{ij} > 0, \tag{7}$$

and thus

$$H_i(t) = \sum_{j=0}^{3} P_{ij} F_{ij}(t) = \sum_{j=0}^{3} Q_{ij}(t). \tag{8}$$

It can be shown (Ross [1]) that

$$P_{ij}(t) = \delta_{ij}\left[1 - H_i(t)\right] + \sum_{k=0}^{3} \int_0^t P_{kj}(t-x)\, dQ_{ik}(x), \tag{9}$$

where δ_{ij} is the Kroneker delta.

If we let $\overline{H}_i(t) = 1 - H_i(t)$ and $q_{ik} * P_{kj}(t)$ indicate the convolution in (9), we obtain

$$P_{ij}(t) = \delta_{ij}\overline{H}_i(t) + \sum_{k=0}^{3} q_{ik} * P_{kj}(t), \quad i,j = 0,1,2,3. \tag{10}$$

This expression can also be written in matrix form as

$$\mathcal{P}(t) = \bar{\mathcal{H}}_D(t) + \mathcal{g} * \mathcal{P}(t)$$

where $\bar{\mathcal{H}}_D(t)$ is a diagonal matrix of $\bar{H}_i(t)$ and $\mathcal{g} * \mathcal{P}(t)$ is the matrix convolution with ijth element

$$\left[\mathcal{g} * \mathcal{P}(t)\right]_{ij} = \sum_{k=0}^{3} \int_0^t P_{kj}(t-x)\, dQ_{ik}(x) .$$

We can also define

$$\mu_i = \int_0^\infty t \; dH_i(t) \tag{12}$$

as the expected amount of time spent in state i during each visit; and

$$\eta_{ij} = \int_0^\infty t \; dF_{ij}(t) \tag{13}$$

as the expected amount of time spent in i during each visit, given that the next state entered is j.

It follows that

$$\mu_i = \sum_{j=0}^{3} P_{ij}\,\eta_{ij} . \tag{14}$$

III. TRANSIENT RESULTS

We first examine the transient results. In order to do so, we assume that the transmission and idle times have general distributions, while failure and repair times have exponential distributions. That is

$$T = \text{transmission time} \sim T(t)$$

$$I = \text{idle time} \sim I(t)$$

$$X = \text{time to failure} \sim 1-e^{-\lambda t}$$

$$R = \text{repair time} \sim 1-e^{-\mu t}$$

We first compute $\mathcal{P} = \left[P_{ij} \right]$, and find that

$$P_{01} = P\left[\begin{matrix}\text{next transition will be} \\ \text{into state 1}\end{matrix} \;\middle|\; \begin{matrix}\text{process just} \\ \text{entered state 0}\end{matrix} \right]$$

$$= P\left[\text{failure time} > \text{transmission time} \right]$$

$$= P\ (X > T) = \int_0^\infty e^{-\lambda x}\, dT(x) = \tilde{t}(\lambda).$$

Also

$$P_{10} = P\left[\begin{matrix}\text{next transition will be} \\ \text{into state 0}\end{matrix} \;\middle|\; \begin{matrix}\text{process just} \\ \text{entered state 1}\end{matrix} \right]$$

$$= P\left[\text{failure time} > \text{transmission time} \right]$$

$$= P\ (X > I) = \int_0^\infty e^{-\lambda x}\, dI(x) = \tilde{i}(\lambda).$$

where $\tilde{t}(\lambda)$ and $\tilde{i}(\lambda)$ are the Laplace-Stieljes transforms of the transmission and idle times distributions evaluated at λ, the failure rate.

Now $\mathcal{P} = \left[P_{ij} \right]$ is of the following form

$$\mathcal{P} = \begin{bmatrix} 0 & \tilde{t}(\lambda) & 1-\tilde{t}(\lambda) & 0 \\ \tilde{t}(\lambda) & 0 & 0 & 1-\tilde{t}(\lambda) \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

We need to find $\mathcal{F}(t) = \left[F_{ij}(t)\right]$ as defined in (4).

Now,

$$F_{01}(t) = P[T<t \mid T<X] = \frac{P[T<t, T<X]}{P[T<X]}.$$

In order to determine the joint probability distribution of X and T we "condition" in the value of X.

$$P[T<t, T<X \mid X=x] = \begin{cases} P[T<t] & \text{if } x \geq t \\ P[T<x] & \text{if } x<t, \end{cases}$$

and then

$$\begin{aligned} P[T<t, T<X] &= \int_0^t T(x)\,\lambda e^{-\lambda x}\,dx + \int_t^\infty T(t)\,\lambda e^{-\lambda x}\,dx \\ &= \int_0^t e^{-\lambda x}\,dT(x), \end{aligned}$$

so

$$F_{01}(t) = \frac{\int_0^t e^{-\lambda x}\,dT(x)}{\tilde{t}(\lambda)}.$$

Following the same procedure we find

$$F_{02}(t) = \frac{1 - \bar{T}(t)\, e^{-\lambda t} - \int_0^t e^{-\lambda x}\, dT(x)}{1 - \tilde{t}(\lambda)} ,$$

$$F_{10}(t) = \frac{\int_0^t e^{-\lambda x}\, dI(x)}{\tilde{i}(\lambda)} ,$$

$$F_{13}(t) = \frac{1 - \bar{I}(t)\, e^{-\lambda t} - \int_0^t e^{-\lambda x}\, dI(x)}{1 - \tilde{i}(\lambda)} .$$

We will now determine $\mathcal{Q}(t) = \left[Q_{ij}(t) \right]$ by using the relation defined in (7); i.e.,

$$F_{ij}(t) = \frac{Q_{ij}(t)}{P_{ij}} .$$

This gives us

$$Q_{01}(t) = \int_0^t e^{-\lambda x}\, dT(x) ,$$

$$Q_{02}(t) = 1 - \bar{T}(t)\, e^{-\lambda t} - \int_0^t e^{-\lambda x}\, dT(x) ,$$

$$Q_{10}(t) = \int_0^t e^{-\lambda x}\, dI(x) ,$$

$$Q_{13}(t) = 1 - \bar{I}(t)\, e^{-\lambda t} - \int_0^t e^{-\lambda x}\, dI(x) ,$$

$$Q_{20}(t) = Q_{31}(t) = R(t) = 1 - e^{-\mu t} ,$$

$$Q_{03}(t) = Q_{12}(t) = Q_{21}(t) = Q_{22}(t) = Q_{23}(t) = Q_{30}(t) = \cdots$$

$$\cdots = Q_{32}(t) = Q_{33}(t) = 0 .$$

By means of the relation (8) we find that

$$\bar{H}_0(t) = \bar{T}(t)\, e^{-\lambda t},$$
$$\bar{H}_1(t) = \bar{I}(t)\, e^{-\lambda t},$$
$$\bar{H}_2(t) = \bar{R}(t),$$
$$\bar{H}_3(t) = \bar{R}(t).$$

We now have the necessary elements to compute the relationship stated in (11); that is,

$$\mathcal{P}(t) = \bar{H}_D(t) + g * \mathcal{P}(t)$$

to determine $\mathcal{P}(t)$.

Taking Laplace transforms and rewriting (11) we get

$$\tilde{\mathcal{P}}(s) = \tilde{\bar{H}}_D(s) + \tilde{g}(s)\, \tilde{\mathcal{P}}(s)$$

which can also be written

$$\tilde{\mathcal{P}}(s) = \left[I - \tilde{g}(s) \right]^{-1} \tilde{\bar{H}}_D(s)$$

Now we have that

$$\tilde{g}(s) = \begin{bmatrix} 0 & \tilde{t}(\lambda+s) & \frac{\lambda}{\lambda+s}\left[1 - \tilde{t}(\lambda+s)\right] & 0 \\ \tilde{i}(\lambda+s) & 0 & 0 & \frac{\lambda}{\lambda+s}\left[1 - \tilde{i}(\lambda+s)\right] \\ \tilde{r}(s) & 0 & 0 & 0 \\ 0 & \tilde{r}(s) & 0 & 0 \end{bmatrix}$$

where, for instance, $q_{02}(s)$ is determined by

$$\tilde{q}_{02}(s) = \int_0^\infty e^{-\lambda s}\, dQ_{02}(t) = \int_0^\infty e^{-\lambda s}\, d\left[1 - e^{-\lambda t}\bar{T}(t) - \int_0^t e^{-\lambda x}\, dT(x)\right]$$

$$= \int_0^\infty e^{-\lambda s}\left[e^{-\lambda t} - T(t)\, e^{-\lambda t}\right] dt$$

$$= \lambda \int_0^\infty e^{-(\lambda+s)t}\, dt + \lambda \int_0^\infty e^{-(\lambda+s)t}\, T(t)\, dt$$

$$= \frac{\lambda}{\lambda+s} - \lambda \tilde{T}(\lambda+s),$$

where $\tilde{T}(\lambda+s)$ is the Laplace transform of the distribution of transmission time, and we know that

$$\tilde{T}(\lambda+s) = \frac{\tilde{t}(\lambda+s)}{\lambda+s},$$

so then

$$\tilde{q}_{02}(s) = \frac{\lambda}{\lambda+s}\left[1 - \tilde{t}(\lambda+s)\right]$$

In our problem we are only interested in $\mathcal{P}_{00}(t)$; i.e., the probability that a transmitter is in a transmission period at time t. Then we only need to determine $\tilde{\mathcal{P}}_{00}(s)$. But $\tilde{\tilde{\mathcal{P}}}_{00}(s)$ is the (0,0) element of $\left[I-\tilde{q}(s)\right]^{-1}\tilde{\tilde{H}}_D(s)$ which requires only the (0,0) element of $\left[I-\tilde{q}(s)\right]^{-1}$. Because of the special structure of q(s) algebraic inversion to obtain this one element was possible. After much algebra the result is

$$\tilde{\mathcal{P}}_{00}(s) = \frac{1 - \frac{\lambda}{\lambda+s}\left[1-\tilde{i}(\lambda+s)\right]\tilde{r}(s)}{\left\{1 - \frac{\lambda}{\lambda+s}\left[1-\tilde{t}(\lambda+s)\right]\tilde{r}(s)\right\}\left\{1 - \frac{\lambda}{\lambda+s}\left[1-\tilde{i}(\lambda+s)\right]\tilde{r}(s)\right\}\cdots}$$

$$\cdots \overline{\quad - \tilde{i}(\lambda+s)\,\tilde{t}(\lambda+s)\quad}\,.$$

Two examples are given to illustrate the transient results.

Example 1

Let transmission time be exponential with mean α and idle time also exponential with mean β. Then

$$\tilde{P}_{\infty}(s) = \frac{\mu^2/\beta + (\mu^2 + 2\mu/\beta + \lambda\mu)s + (2\mu + 1/\beta + \lambda)s^2}{s\{\mu(1/\alpha + 1/\beta)(\lambda+\mu) + [(\lambda+\mu)^2 + (2\mu+\lambda)(1/\alpha + 1/\beta)]s} \cdots$$

$$\cdots \frac{+ s^3}{+ [2(\lambda+\mu) + 1/\alpha + 1/\beta]s^2 + s^3\}} .$$

This can be written as

$$\tilde{P}_{oo}(s) = \frac{(s+a)(s+b)(s+c)}{s(s+d)(s+f)(s+g)} \tag{15}$$

where a, b and c must satisfy the relations

$$abc = \mu^2/\beta ,$$
$$c(a+b)+ab = \mu^2 + 2\mu/\beta + \lambda\mu ,$$
$$a+b+c = 2\mu + 1/\beta + \lambda .$$

Also d, f and g must satisfy

$$dfg = \mu(1/\beta + 1/\alpha)(\lambda+\mu) ,$$
$$d(f+g)+fg = (\lambda+\mu)^2 + (2\mu+\lambda)(\tfrac{1}{\beta} + 1/\alpha) ,$$
$$d+f+g = 2(\lambda+\mu) + 1/\alpha + 1/\beta .$$

Then the inverse of the Laplace transform in (15) will be

$$P_{oo}(t) = A + B(1-\tfrac{a}{d})e^{-dt} + C(1-\tfrac{a}{f})e^{-ft} + D(1-\tfrac{a}{g})e^{-gt}$$

where

$$A = \frac{abc}{dfg},$$

$$B = \frac{d^2-(b+c)d+bc}{(f-d)(g-d)},$$

$$C = \frac{f^2-(b+c)f+bc}{(d-f)(g-f)},$$

$$D = \frac{g^2-(b+c)g+bc}{(d-g)(e-g)}.$$

If we let

$$\alpha = 2.0 \text{ sec.}$$

$$\beta = 20.0 \text{ sec.}$$

$$1/\lambda = 180.0 \text{ sec.}$$

$$1/\mu = 30.0 \text{ sec.}$$

then,

$$P_{00}(t) = .078 + .001e^{-.033t} + .013e^{-.039t} + .908e^{-.556t}.$$

Figure 2 illustrates $P_{00}(t)$ for different values of t. It can be observed that for the given parameters values, the system reaches the steady state very fast.

Besides the method just described to take the inverse of $\tilde{P}_{00}(s)$, numerical inversion (LINV IBM-360) was used and yielded the same results for $P_{00}(t)$ using different values of t.

Table I shows L(t), the probability that a message transmitted at time t is lost, for different values of t and for n = 30 and r = 3.

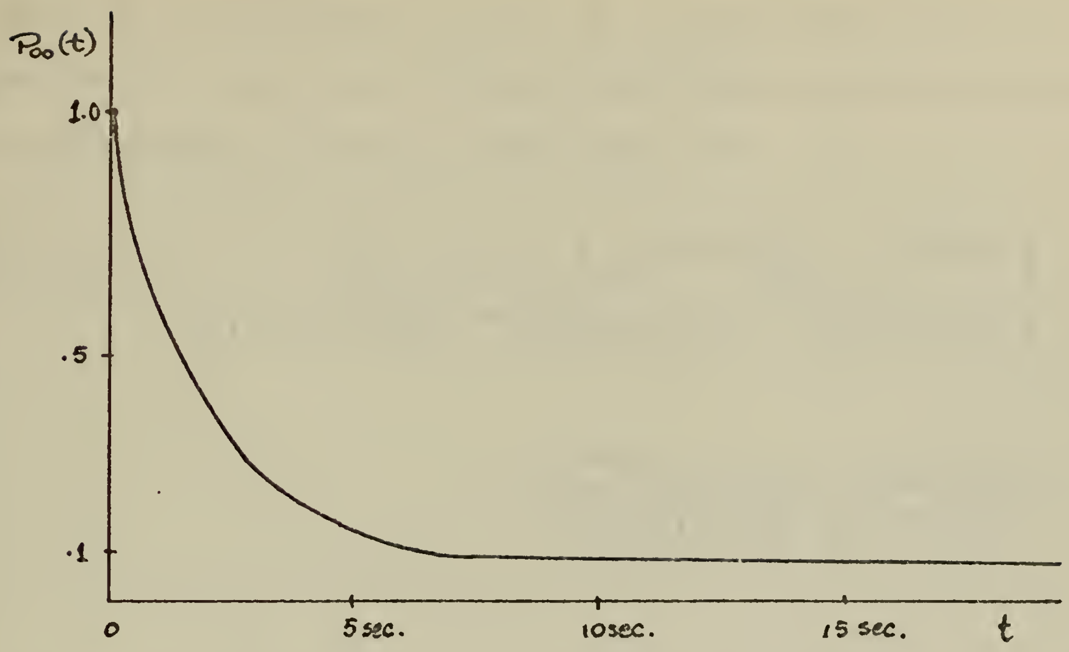

Figure 2. The probability that a transmitter is in a transmission period plotted as a function of t.

t	L(t)
0 sec.	1.0
2 "	0.99
3 "	0.99
4 "	0.97
5 "	0.85
6 "	0.68
7 "	0.49
8 "	0.35
9 "	0.30
10 "	0.28

Table I. Probability that a message transmitted at time t is lost.

Example 2

Let α be the transmission time and β the idle period length, both assumed constant, and failure and repair times have exponential distributions with parameters λ and μ respectively. Then

$$\tilde{P}_{00}(s) = \frac{\left[(\lambda+\mu+s)s + \lambda\mu e^{-(\lambda+s)\beta}\right]\left[1 - e^{-(\lambda+s)\alpha}\right]}{\left[(\lambda+\mu+s)s + \lambda e^{-(\lambda+s)\beta}\mu\right]\left[(\lambda+\mu+s)s + \lambda\mu e^{-(\lambda+s)\alpha}\right]} \cdots$$

$$\cdots \frac{(\mu+s)}{-\left[(\lambda+\mu)(\mu+s)\right]^2 e^{-(\lambda+s)(\alpha+\beta)}} .$$

Using numerical inversion did not give accurate results because of the existence of sharp peaks in $P_{00}(t)$ due to the constant transmission and idle times assumed. Exact inversion in this case was not found to be possible.

IV. STEADY STATE RESULTS

We now examine the output of the model after a long period of time; that is, in the steady state.

If we denote by J_o the initial state of the process, and for $n \geq 1$ let J_n denote the state of the process immediately after the n^{th} transition has occurred, the process $\{J_n, n = 0,1,2,\ldots\}$ is an embedded Markov Chain with transition probabilities $P_{ij} = Q_{ij}(\infty)$. Furthermore, if the embedded Markov Chain is aperiodic and irreducible, defining

$$P_j = \lim_{t \to \infty} P[Z(t) = j \mid Z(0) = i]$$

it can be shown (Ross [1]) that

$$P_j = \frac{\pi_j \mu_j}{\sum_{i=0}^{3} \pi_i \mu_i}, \qquad (16)$$

where π_j, j = 0,1,2,3 are the limiting probabilities for the embedded Markov Chain. That is, if $\Pi = (\pi_0 \pi_1 \pi_2 \pi_3)$, then the π_j's are the solutions for $\Pi = \Pi \cdot P$, $\sum_{j=0}^{3} \pi_j = 1$.

The same examples used to illustrate the transient results will be used here.

On example 1 transmission and idle times have both exponential distributions with parameters $1/\alpha$ and $1/\beta$ respectively. The matrix P is now given by

$$\mathcal{P} = \begin{bmatrix} 0 & \dfrac{1}{1+\alpha\lambda} & 1-\dfrac{1}{1+\alpha\lambda} & 0 \\ \dfrac{1}{1+\beta\lambda} & 0 & 0 & 1-\dfrac{1}{1+\beta\lambda} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

This $\mathcal{P}$ matrix is of a chain with period 2 so is not aperiodic. However the equations $\Pi = \Pi \cdot \mathcal{P}$, $\sum_{j=0}^{3} \pi_j = 1$ can still be solved in which case the π_j's are interpreted to be the probability of being in a given state at an arbitrary time point. If one knew that the time point were an odd or even number of periods after the process started then the probability of being in a given state would not be given by the π_j's. However, such a situation is not of interest to us in this paper and the π_j's are in fact the numbers which we want.

Using this matrix of transition probabilities we arrive at

$$\pi_0 = \frac{1+\alpha\lambda}{D},$$

$$\pi_1 = \frac{1+\beta\lambda}{D},$$

$$\pi_2 = \frac{\alpha\lambda}{D},$$

$$\pi_3 = \frac{\beta\lambda}{D},$$

where $D = 2[1 + (\alpha+\beta)\lambda]$.

Recalling that we defined the expected amount of time spent in state i during each visit, given that the next state entered is j as

$$\eta_{ij} = \int_0^\infty t \, dF_{ij}(t)$$

we have

$$\eta_{01} = \frac{1}{1/\alpha + \lambda} ,$$

$$\eta_{02} = \frac{1}{1/\alpha + \lambda} ,$$

$$\eta_{10} = \frac{1}{1/\beta + \lambda} ,$$

$$\eta_{13} = \frac{1}{1/\beta + \lambda} ,$$

$$\eta_{20} = \eta_{31} = 1/\mu .$$

The expected amount of time spent in each of the states in each visit, $\mu_i = \sum_{i=0}^{3} P_{ij} \eta_{ij}$, is

$$\mu_0 = \frac{1}{1/\alpha + \lambda} ,$$

$$\mu_1 = \frac{1}{1/\beta + \lambda} ,$$

$$\mu_2 = \mu_3 = 1/\mu .$$

It is possible now to compute P_j, j = 0,1,2,3 by using the results just derived and the relation stated in (16).

On example 2 we let α and β be the transmission and idle times respectively, assumed constant. We also let failure and repair times have exponential distributions with parameters λ and μ respectively. The matrix $\mathcal{P} = \left[P_{ij} \right]$ will then be

$$\mathcal{P} = \begin{bmatrix} 0 & e^{-\alpha\lambda} & 1-e^{-\alpha\lambda} & 0 \\ e^{-\beta\lambda} & 0 & 0 & 1-e^{-\beta\lambda} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Using the same procedure as in example 1, we can find the necessary elements to solve for the P_j's.

Table II shows the values of the P_j's, $j = 0,1,2,3$ computed for examples 1 and 2 for $\alpha = 2.0$, $\beta = 20.0$, $1/\lambda = 180.0$ and $1/\mu = 30.0$ seconds.

State j	Probabilities P_j	
	Example 1.	Example 2.
0	0.0779222	0.1109148
1	0.7792229	0.8888418
2	0.0129867	0.0000211
3	0.1298679	0.0002221

Table II. Steady state probabilities for examples 1 and 2.

We can now compute $L(\infty)$, the steady state probability that a message is lost, by means of the relation stated in Part II.

Table III shows the values of $L(\infty)$ for different values of n and r for examples 1 and 2.

	Example 1.				Example 2.			
n \ r	2	3	4	5	2	3	4	5
20	.21	.07	.02	.004	.38	.13	.04	.02
30	.43	.21	.09	.03	.66	.42	.23	.10
40	.63	.39	.21	.09	.83	.65	.45	.27
50	.77	.57	.37	.20	.92	.81	.66	.47

Table III. Steady state probability that a message is lost computed for examples 1 and 2.

The case n = 30 is shown plotted in figure 3 for different values of the number of receivers. For both examples we see that the loss probability decreases rapidly with an increase in the number of receivers, and for r = 5 the loss probabilities decrease to .03 and .10 for examples 1 and 2 respectively.

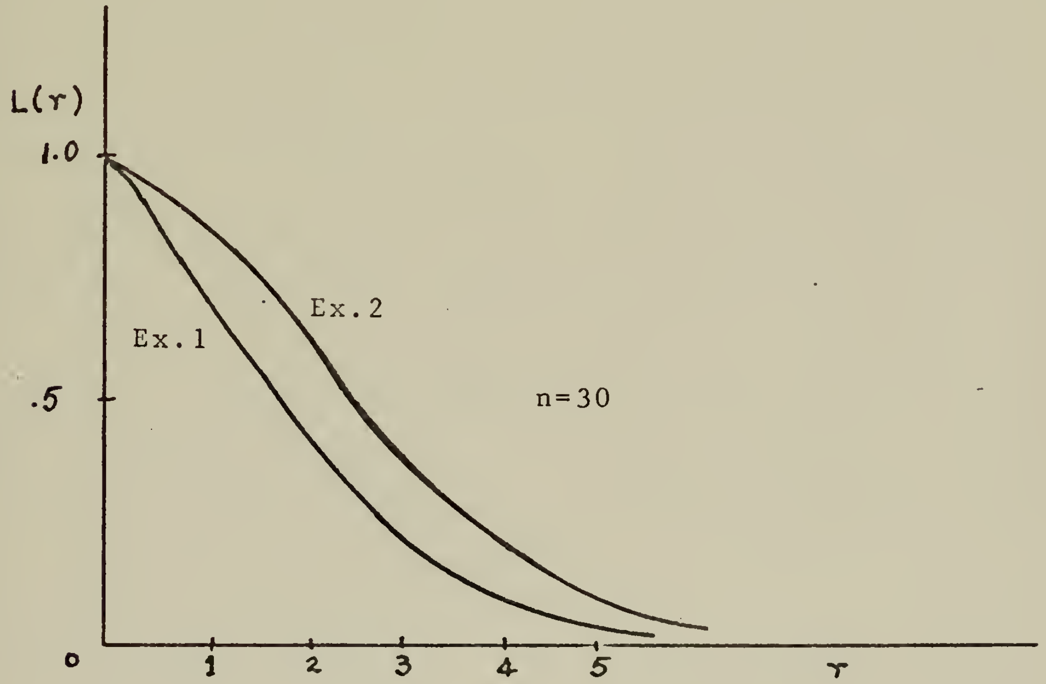

Figure 3. Probability of a lost message as a function of the number of receivers.

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A communications system composed of n transmitters and r receivers and which is subject to failure is modeled as a semi-Markov process having four states. Transient and steady state results are analyzed to determine the probability that the system is in any given state. Two examples are given to illustrate the results in the transient as well as in the steady state.

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